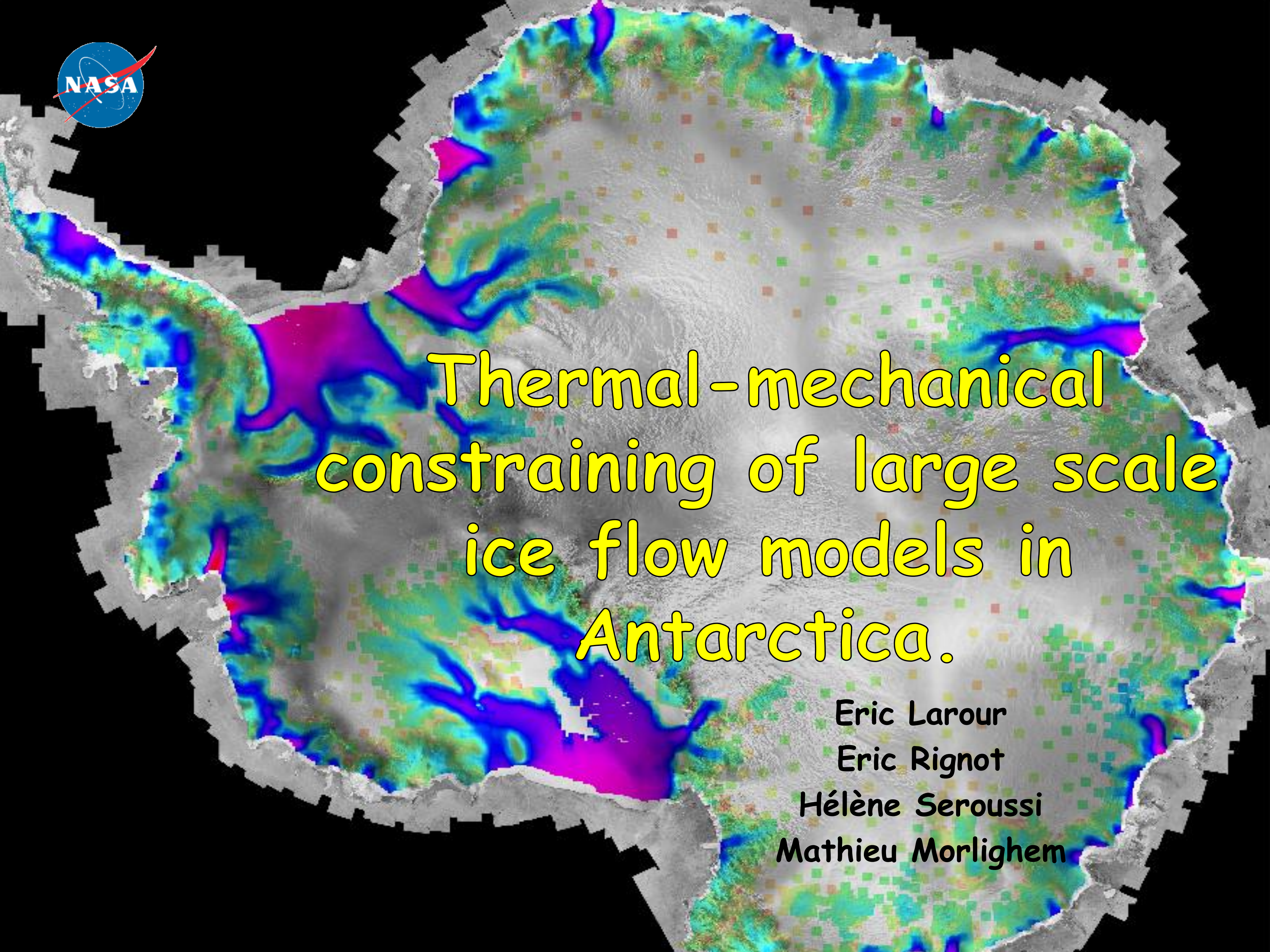

# Thermal-mechanical constraining of large scale ice flow models in Antarctica.

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# Outline

1. Introduction
2. Higher order inverse control methods.
3. Large scale modeling using Anisotropic Mesh Adaptation.
4. Ice flow model of Antarctica using ISSM.
5. Perspectives

# 1 Introduction

- Large scale modeling of Antarctica:
  - 1km resolution on Antarctica -> 20 Million elements in 2d
  - 400 million in 3d (20 vertical layers)
  - Full stokes: 1.6 billion dofs. (4 per node)
  - Cost is prohibitive.
- Constraints on bedrock friction and ice rheology:
  - Paleo runs for large scale models are hard to converge to present time.
  - Paleo runs usually do not account for full stress equilibrium (SIA).
  - A mix of paleo run and inverse control methods at present time could be necessary (similar to GCM spin up).

# 2 Higher order inverse control methods.

- Cost function: $$J = \iint_{Surface} \frac{1}{2} \left\{ (u - u_{obs})^2 + (v - v_{obs})^2 \right\} dxdy$$
- We augment J with the ice flow model desired, multiplied by adjoint vectors. The model equations depend on the order modeling desired:
- Macayeal: $$J' = \iint_{S} \frac{1}{2} \left\{ (u - u_{obs})^2 + (v - v_{obs})^2 \right\} dxdy +$$

$$\iint_S \lambda_x(x,y) \left\{ \frac{\partial}{\partial x}\left( 2\nu H \left( 2\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right) + \frac{\partial}{\partial y}\left( \nu H \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right) - \rho g H \frac{\partial z_s}{\partial x} - \beta^2 u \right\} dxdy +$$

$$\iint_S \lambda_y(x,y) \left\{ \frac{\partial}{\partial y}\left( 2\nu H \left( 2\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} \right) \right) + \frac{\partial}{\partial x}\left( \nu H \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right) - \rho g H \frac{\partial z_s}{\partial y} - \beta^2 v \right\} dxdy$$

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- Pattyn: $$J = \iint_{Surface} \frac{1}{2} \left\{ (u - u_{obs})^2 + (v - v_{obs})^2 \right\} dxdy +$$

$$\iint_{Volume} \lambda_x(x,y) \left\{ \frac{\partial}{\partial x}\left(2\nu\left(2\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right) + \frac{\partial}{\partial y}\left(\nu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right) + \frac{\partial}{\partial z}\left(\nu\frac{\partial u}{\partial z}\right) - \rho g \frac{\partial z_s}{\partial x} \right\} dxdy +$$

$$\iint_{Volume} \lambda_y(x,y) \left\{ \frac{\partial}{\partial x}\left(\nu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right) + \frac{\partial}{\partial y}\left(2\nu\left(2\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x}\right)\right) + \frac{\partial}{\partial z}\left(\nu\frac{\partial v}{\partial z}\right) - \rho g \frac{\partial z_s}{\partial y} \right\} dxdy$$

- Second part of misfit is integrated on the volume instead of the surface. Macayeal is thickness integrated, Pattyn is 3d. Drag is a boundary condition for Pattyn, instead of a surface term for MacAyeal.

- Stokes: $$J = \iint_{Surface} \frac{1}{2}\left\{(u-u_{obs})^2 + (v-v_{obs})^2\right\}dxdy +$$

$$\iint_{Volume} \lambda_x(x,y)\left\{\frac{\partial}{\partial x}\left(2\nu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right) + \frac{\partial}{\partial z}\nu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) - \frac{\partial P}{\partial x}\right\}dxdy +$$

$$\iint_{Volume} \lambda_y(x,y)\left\{\frac{\partial}{\partial x}\left(\nu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right) + \frac{\partial}{\partial y}\left(2\nu\frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial z}\nu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) - \frac{\partial P}{\partial y}\right\}dxdy$$

$$\iint_{Volume} \lambda_z(x,y)\left\{\frac{\partial}{\partial x}\left(\nu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\right) + \frac{\partial}{\partial y}\left(\nu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right) + \frac{\partial}{\partial z}\left(2\nu\frac{\partial w}{\partial y}\right) - \frac{\partial P}{\partial y} - \rho g\right\}dxdy$$

$$\iint_{Volume} \lambda_P(x,y)\left\{\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right\}dxdy$$

- Add vertical stress equilibrium + incompressibility equation. Observations misfit still integrated over surface layer.

- Misfit gradients with respect to drag coefficient:

$$u = k N_{eff}{}^{p} \sigma_{drag}{}^{q} \qquad \text{or} \qquad \sigma_{drag} = \alpha^2 u^r N_{eff}{}^{-s}$$ (Paterson, 1994)

- Stokes:

$$\frac{\partial J}{\partial \alpha} = -\lambda_x \left(2\alpha \left(v_x - v_z n_x n_z\right)\right) - \lambda_x \left(2\alpha \left(v_x - v_z n_x n_z\right)\right) \\ -\lambda_z \left(2\alpha \left(-v_x n_x n_z - v_y n_y n_z\right)\right)$$

- Pattyn and MacAyeal:

$$\frac{\partial J}{\partial \alpha} = -2\alpha\left(\lambda_x v_x + \lambda_y v_y\right)$$

- Thermal constraints:
  - Ice rheology inverted for ice shelves (no bedrock friction).
  - For ice sheet: inversion of drag and ice rheology is a severely underconstrained optimization problem -> solution exhibits multiple extrema.

  We run steady-state thermal model at each iteration of the inverse control method, so that thermally induced stresses are accounted for. Thermal model is equally computationally intensive.

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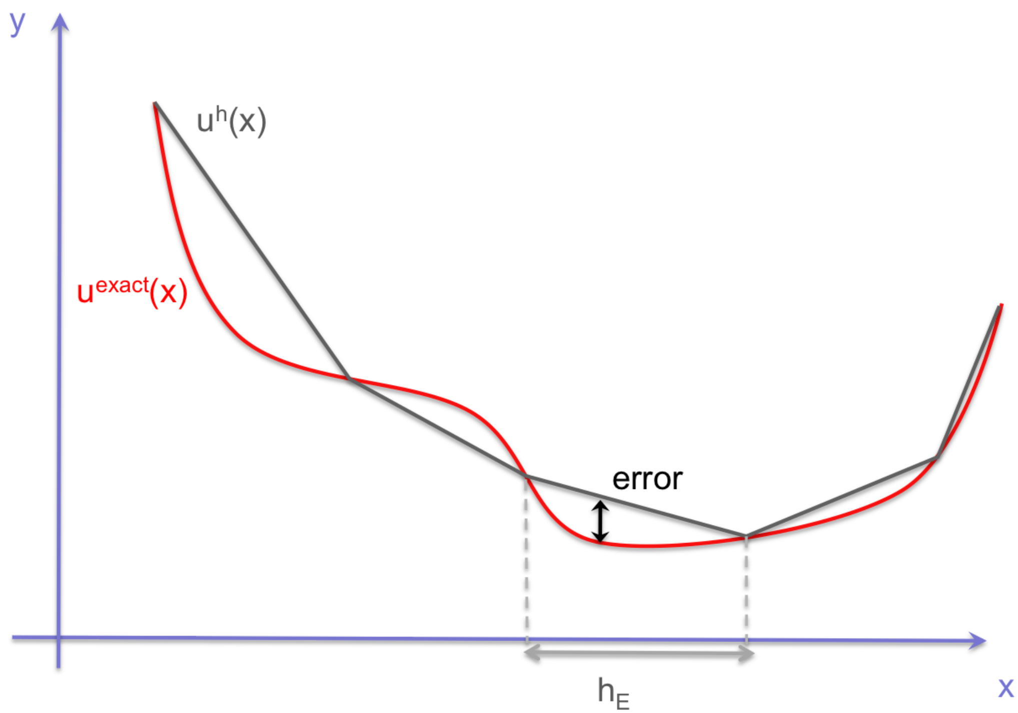
# 3 Large scale modeing using Anisotropic Mesh Adaptation.

- If the solution u(x) is approximated by $u_h(x)$, with piecewise linear interpolation, a local approximation error can be defined over an element E to be :

$$x \in [0; h_E] : error = \left| u^{exact}(x) - u_E^h(x) \right|$$



Generalized error estimate:

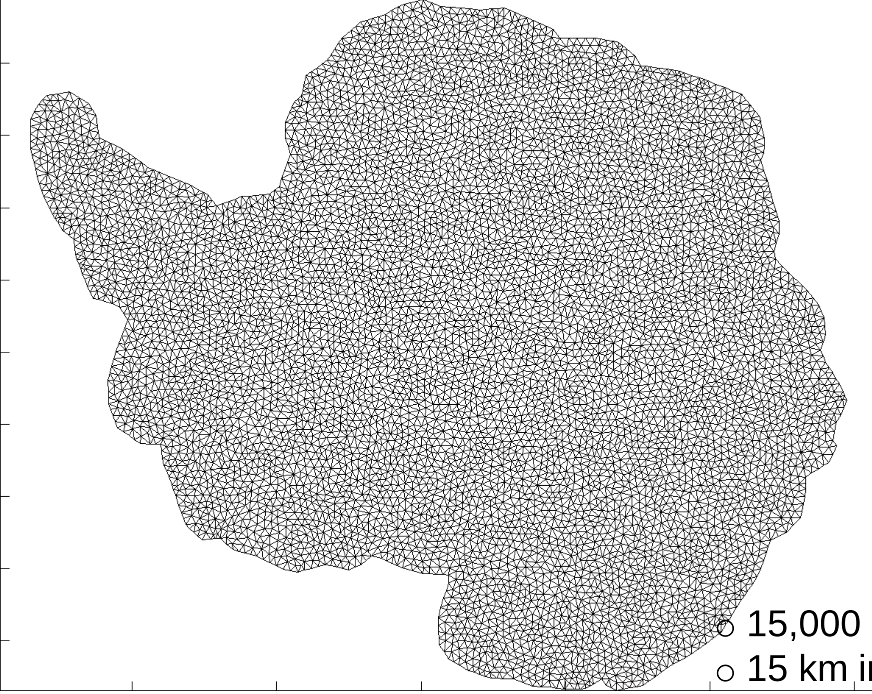
$$\left| u^{exact}(x) - u_E^h(x) \right| \le c_d h_E^2 \sup_{(x,y)\in E} \left| H_u(x,y) \right|$$ (Habashi 2000)

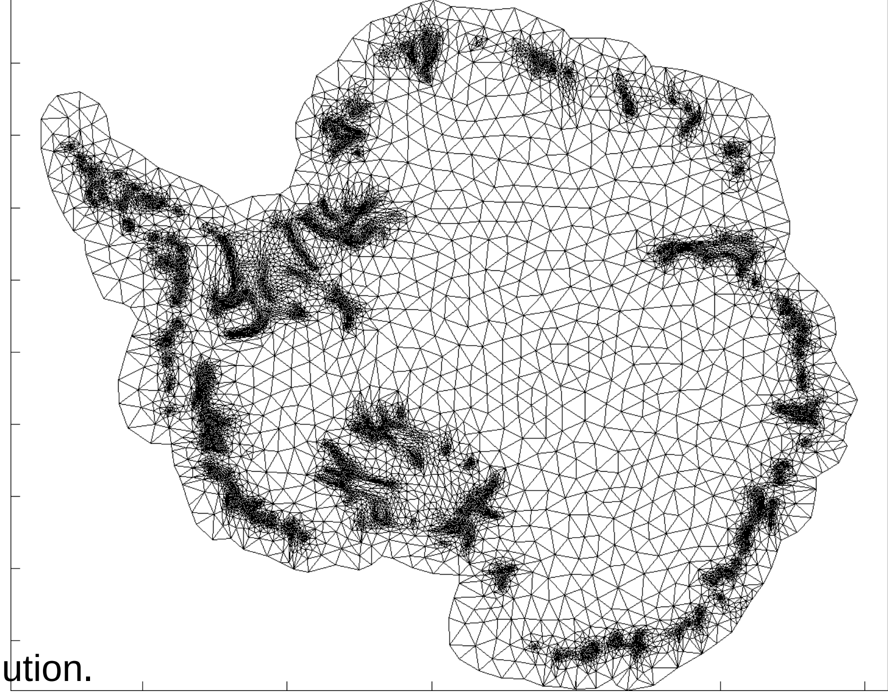
where :

- $h_E$ length of the element edge
- $C_d$ constant that depends only on the space dimension (1.8 in 1d, 2.9 in 2d)
- $H_f$ (x; y) Hessian matrix of u, |Hu(x; y)| its spectral norm

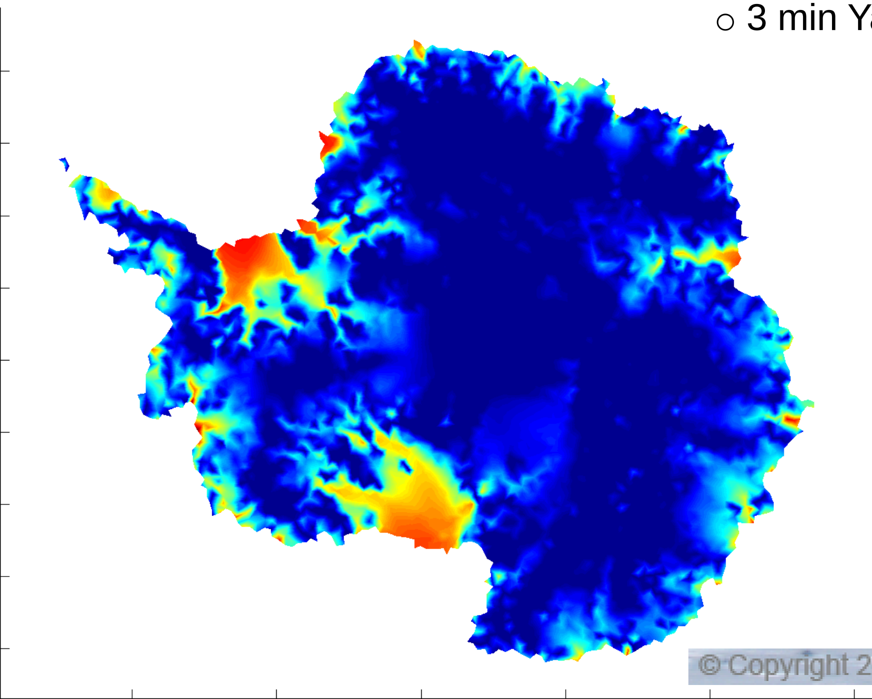
-> use Hessian matrix to minimize the error estimate, by remeshing along principal directions of Hessian matrix, according to eigenvalue magnitude.

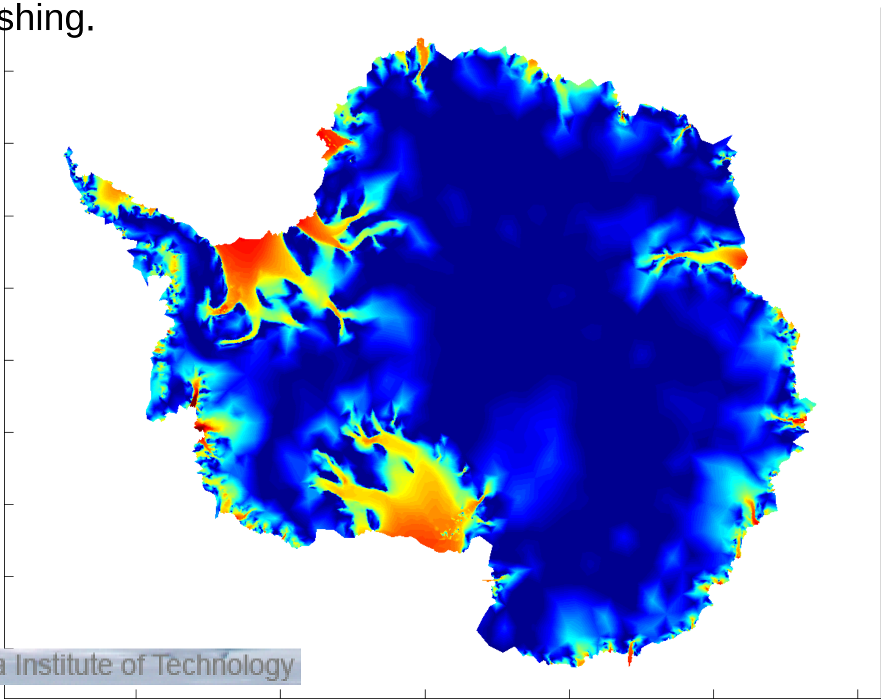
Tool: YAMS, developed within the GAMMA research project at INRIA-Rocquencourt. Anisotropic. Pascal Frey.

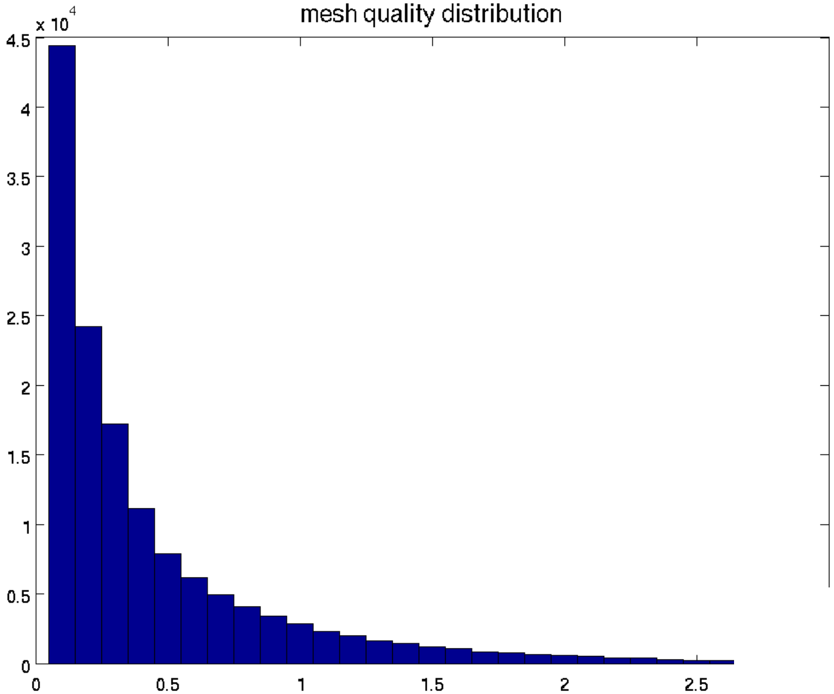




- 15,000 elements.
- 15 km initial resolution.
- 5km final resolution on icestreams.
- 3 min Yams remeshing.

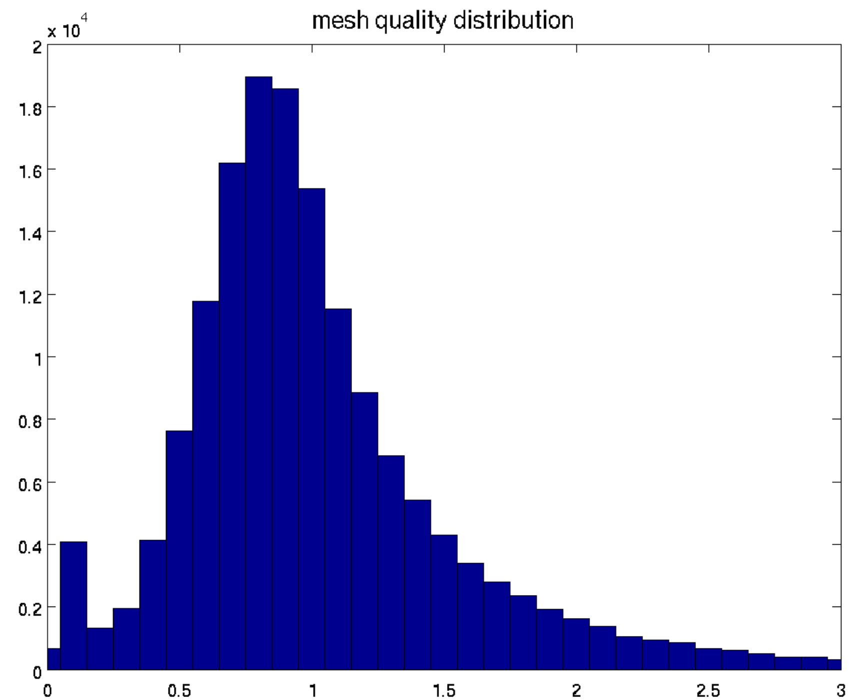







In transformed error coordinates space (along Hessian directions), mesh triangles should tend to be equilateral (best capture of discretization error).

Mesh quality: measure of distortion from equilateral discretization error. Tends to 1 for equilateral triangles in error space.



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# 4 Ice flow model of Antarctica using ISSM.

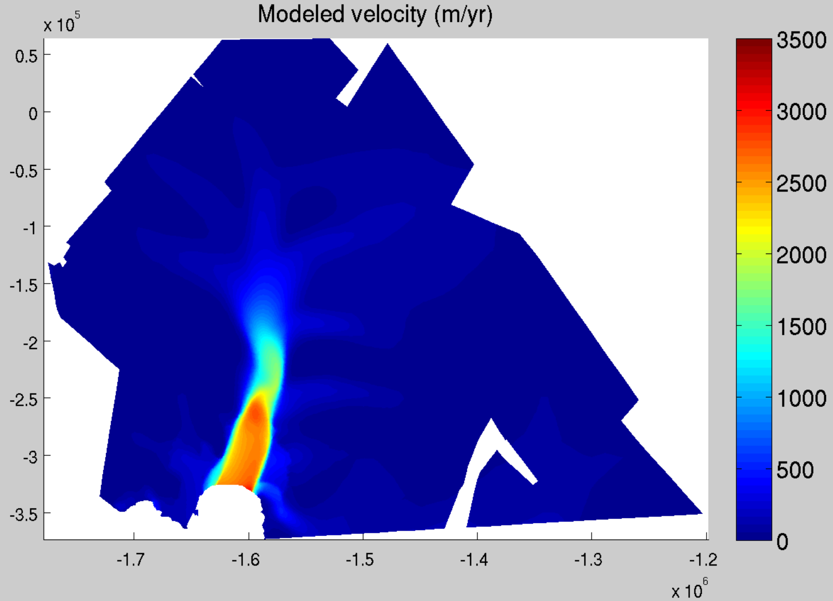
- ISSM: Ice Sheet System Model, developed by JPL's R&TD program, funded by JPL and NASA (Map09).

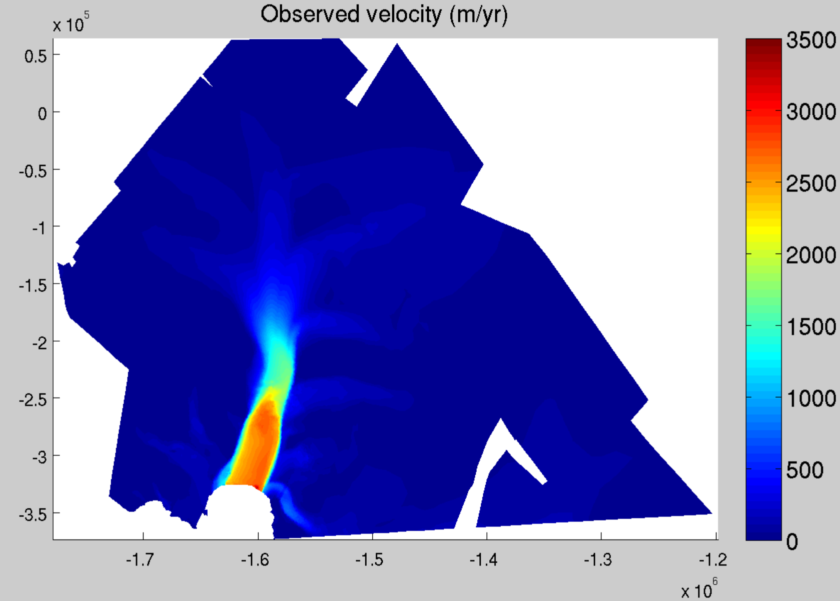
- Large scale model of Antarctica, using anisotropic remeshing:
  - 150,000 2d elements: MacAyeal formulation.
  - 1,200,000 3d elements (8 extrusion layers, distorted towards bedrock). Pattyn formulation.

  Icestreams resolved at 3km, interior of ice sheet captured at 50km.

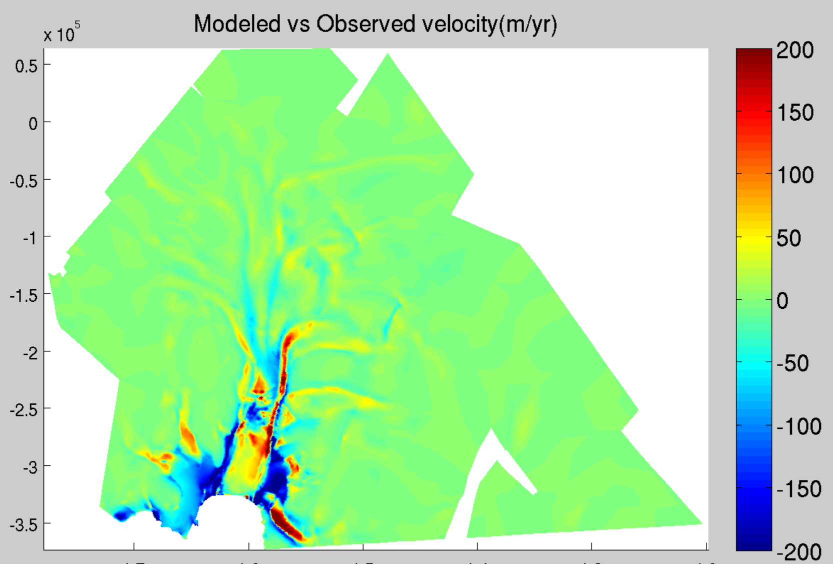
- Diagnostic run, constrained using inverse control methods on drag:
  - Background run (40 iterations) to correctly constrain entire ice sheet.
  - Refinement on all basins (20 basins) to capture icestreams.

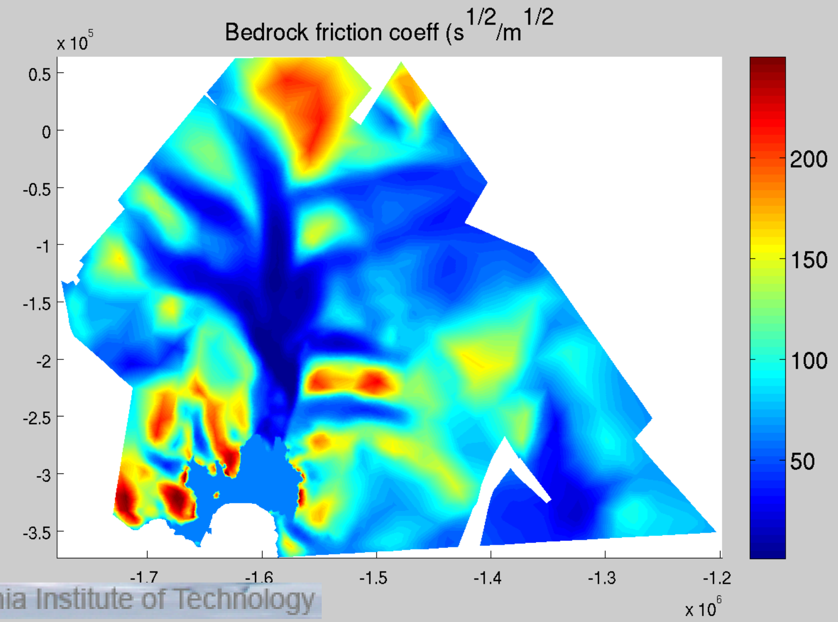
- Firn Layer: van den Broeke, M.R., Towards quantifying the contribution of the Antarctic ice sheet to global sea level change. Journal of Physics. IV France, 2006 (139) 170-187
- Temperatures: Giovinetto, M.B., N.M. Waters, and C.R. Bentley, Dependence of Antarctic surface mass balance on temperature, elevation and distance to open ocean, Journal of Geophysycal Research, 1990, 95, 3517-3531
- Surface: Bamber, Jonathan L., Jose Luis Gomez-Dans, and Jennifer A. Griggs. 2009. *Antarctic 1 km Digital Elevation Model (DEM) from Combined ERS-1 Radar and ICESat Laser Satellite Altimetry*. Boulder, Colorado USA: National Snow and Ice Data Center. Digital media.
- Thickness: Lythe,M.B., D.G. Vaughan and Consortium BEDMAP, BEDMAP: A new ice thickness and subglacial topographic model of Antarctica, Journal of Geophysical Research, 2001, 106 (B6), 11,335-11,352
- Grounding Line, Ice Front, Ice Rises: Rignot unpublished.
- Surface velocity map: Rignot, unpublished.

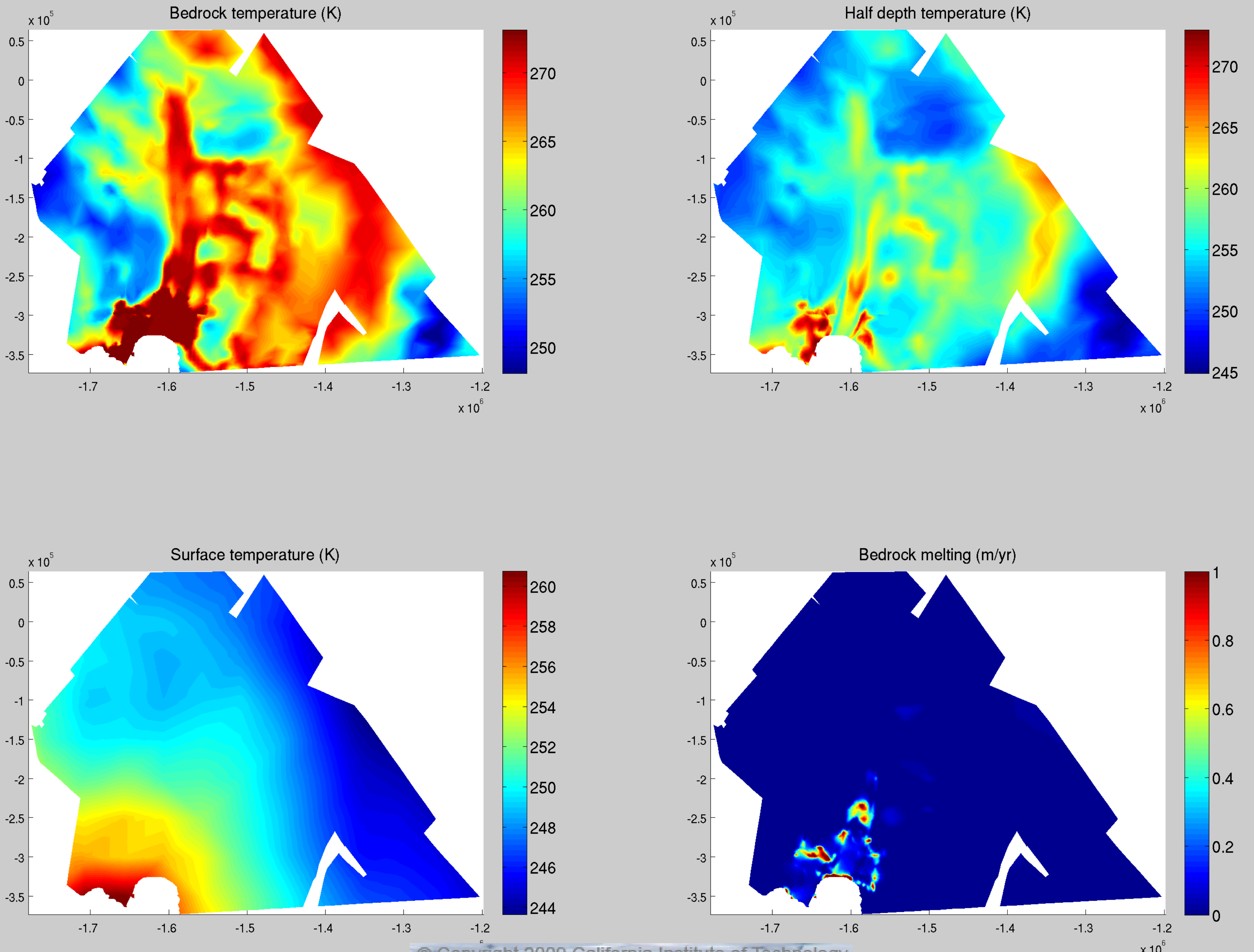


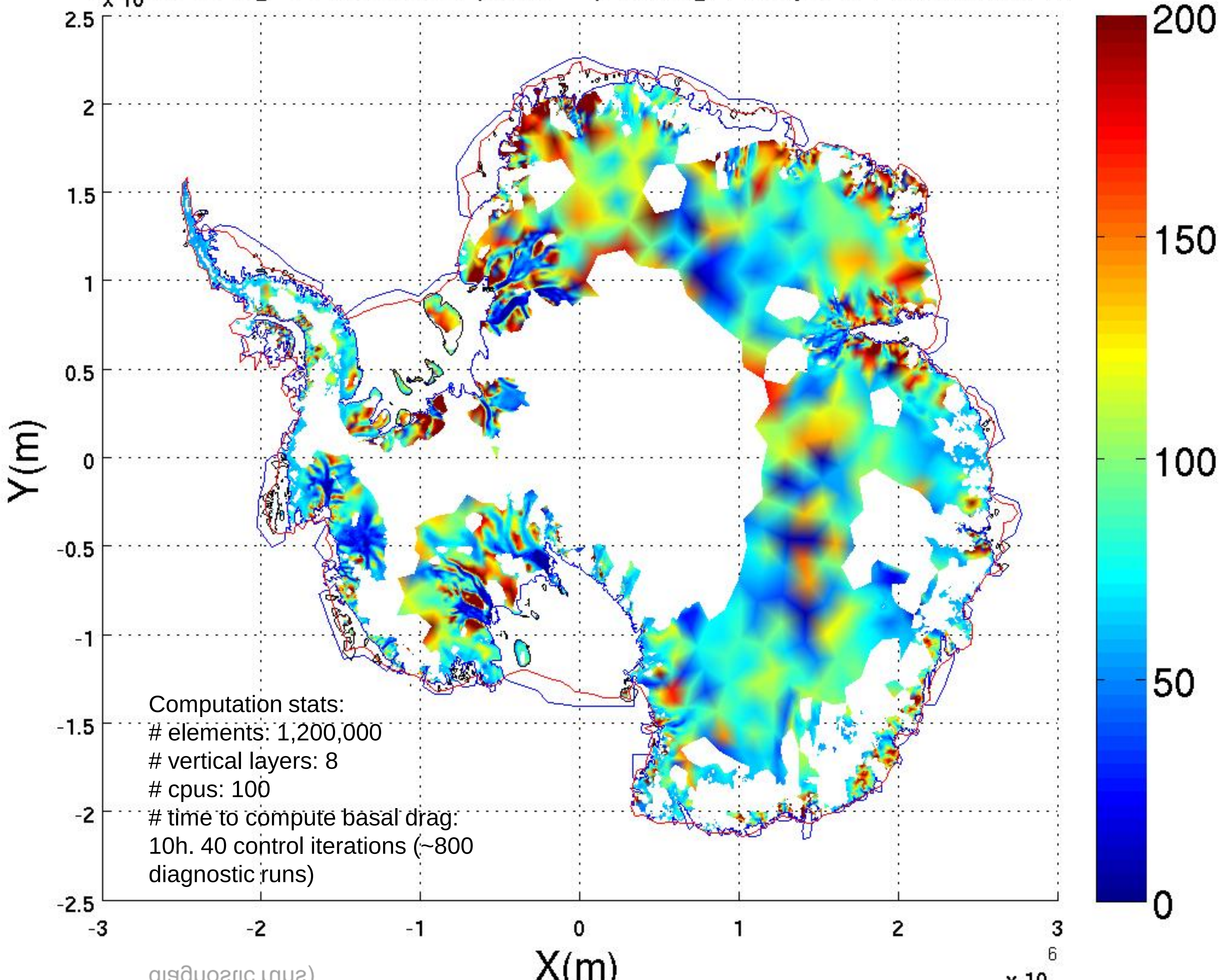

Cf: Mathieu Morlighem poster, Tuesday session.



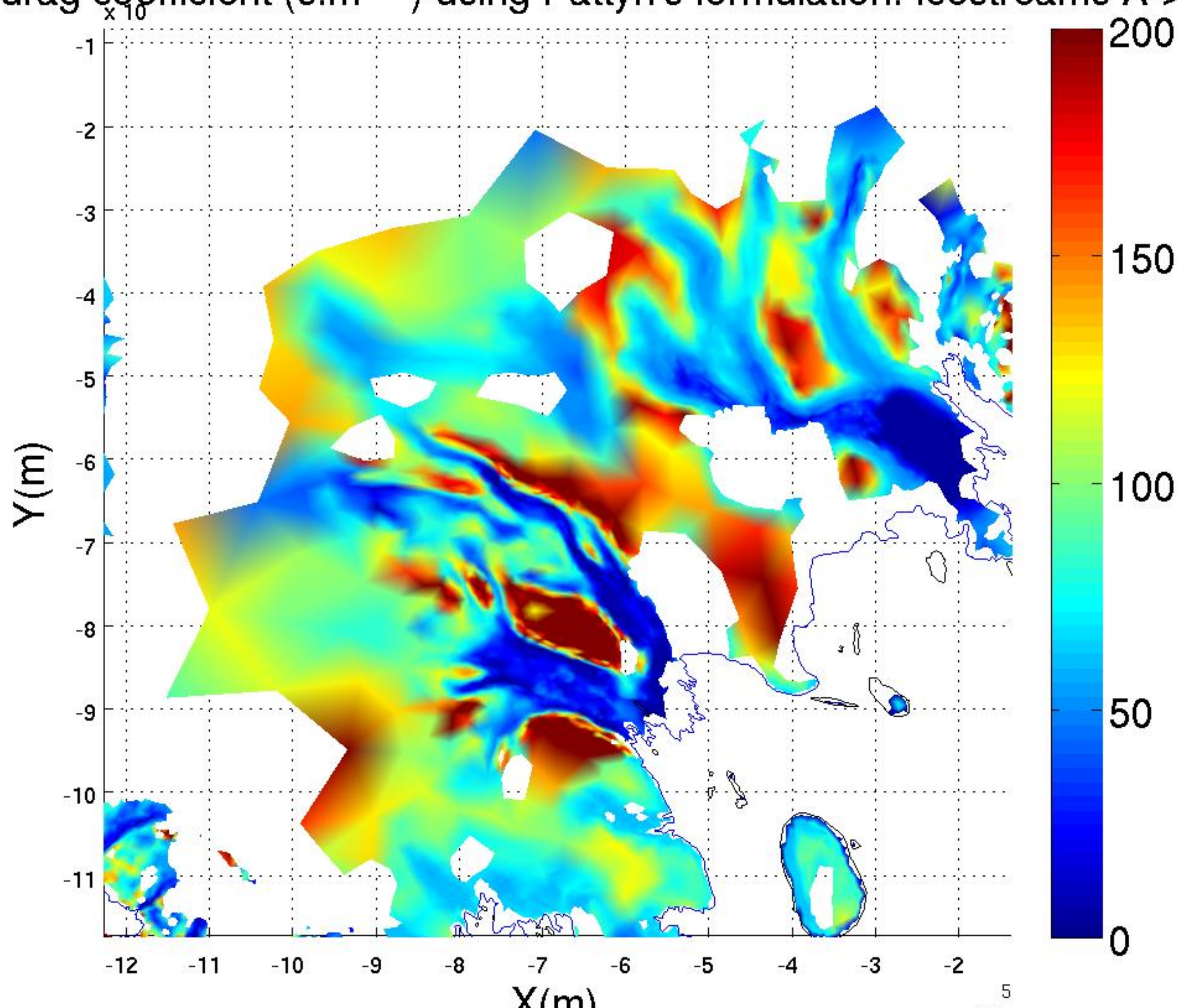




Bedrock temperature (K)
x 10^5
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-1.7
-1.6
-1.5
-1.4
-1.3
-1.2
x 10^6
270
265
260
255
250
Half depth temperature (K)
270
265
260
255
250
245
Surface temperature (K)
260
258
256
254
252
250
248
246
244
Bedrock melting (m/yr)
1
0.8
0.6
0.4
0.2
0

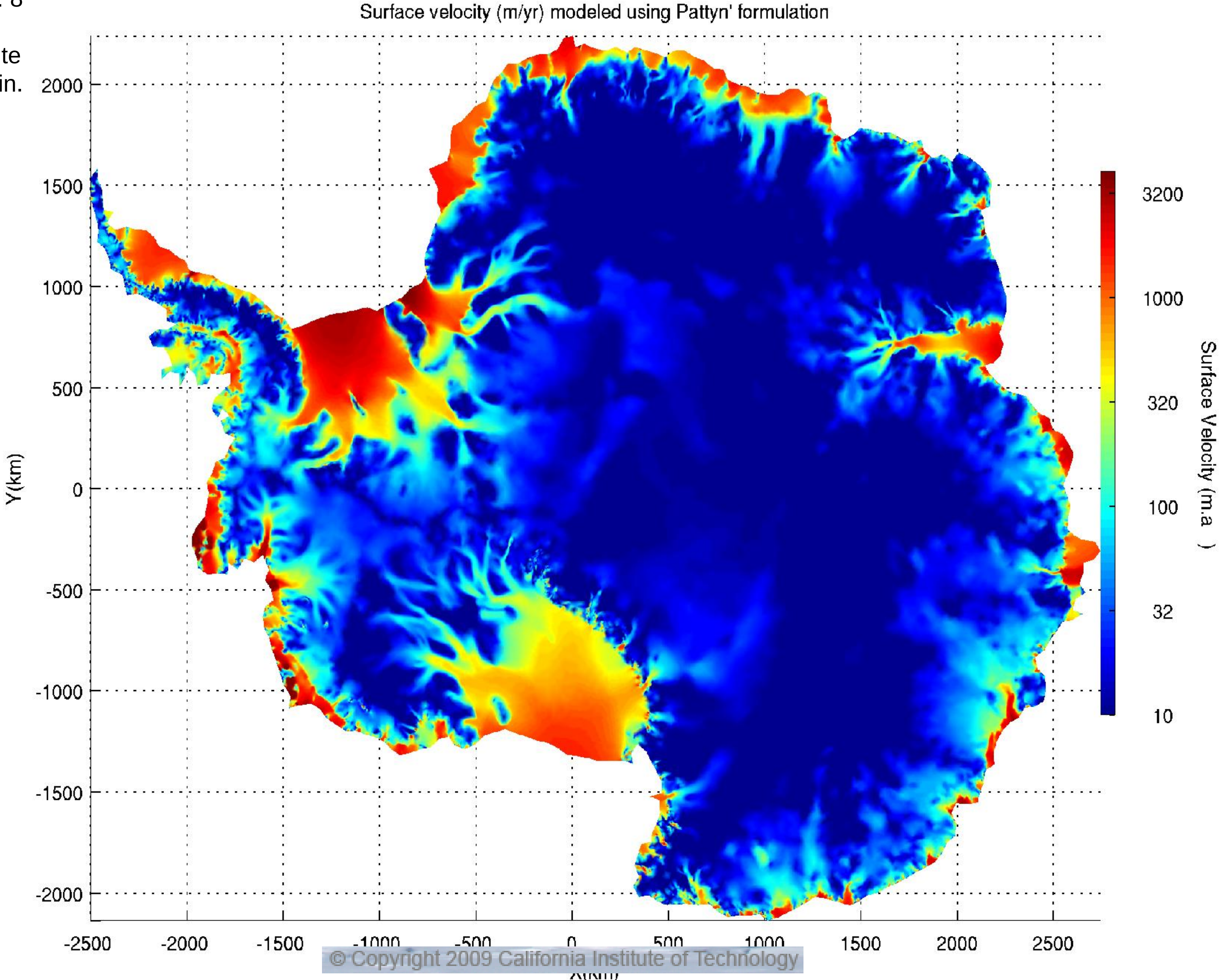
# Basal drag coefficient (s.m$^{-1/2}$) using Pattyn's formulation.



Basal drag coefficient ($s.m^{-1/2}$) using Pattyn's formulation. Icestreams A->E.



Computation stats:
#elements: 1,200,000
#vertical layers: 8
#cpus: 100
#time to compute
diagnostic: 5 min.



# 5 Conclusions and perspectives.

- Higher order inverse control methods are computationally affordable, using adaptative remeshing.
- InSAR data becoming available to constrain entire continent.
- Spin ups can now combine paleo-runs with inverse control methods to constrain Antarctica ice flow.
- ISSM capable of fully constraining present day diagnostics, with assumption of thermal steady-state.
- Embedded Full-Stokes inversion computationally possible in the next couple years.
- Short term transients should be possible with full resolution models.

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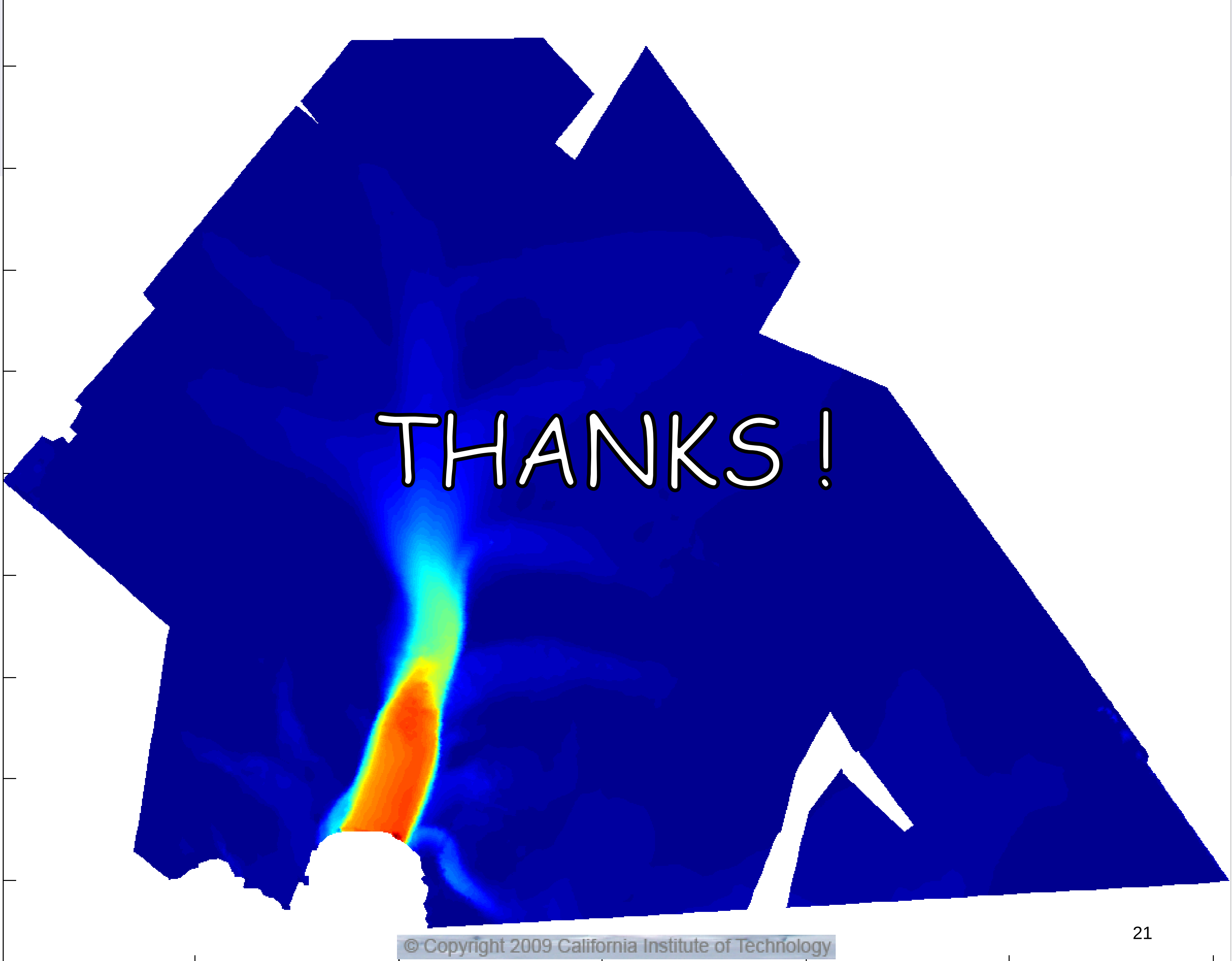
THANKS !